The manner of finding of the true Sum of the Infinite Secants of an Arch, by an Infinite Series.

Which being found and compared with the Sum of the Secants found, by adding of the Secants of whole Minutes, or Centesmes of a Degree, from a Table of Natural Secants, do plainly demonstrate that Mr. *Edward Wright*'s Nautical Planisphere is not a true Projection of the Sphere.

By *RICHARD NORRIS* Mariner.

IN the Year 1599 Mr. *Wright*'s Book Entituled, *The Correction of Errors in Navigation* was printed, in which he saith and takes for granted, that if a Spherical Superficies, with Meridians, Parallels, Rumbs, and the whole Hydrographical Description drawn thereupon, to be inscribed into a Concave Cylinder (their Diameters being equal) in such manner that the Extension of the Spherical Superficies may be equal in every part thereof, as much in Latitude as in Longitude, applying and joyning it self to the Concave Superficies of the Cylinder round about, and all along towards either Pole; each Parallel upon this Spherical Superficies increasing successively from the Æquinoctial towards either Pole, until it come to be of equal Diameter with the Cylinder, and consequently the Meridians, still inclining themselves, until they come to be as far distant, every where each from the other, as they are at the Equinoctial: This Cylindriack Sphere being laid open, maketh a plain Parallellogram, on which all places do stand in the same Longitude, Latitude, Rumb, and Distance, as they were on the Globe.

And because the Spherical Superficies is extended in every part thereof equally, That is as much in Latitude as in Longitude, until it apply'd it self round about the Concavity of the Cylinder; therefore at every point of Latitude in this Planisphere, a part of the Meridian keepeth the same proportion to the like part of the Parallel, that the like part of the Meridian and Parallel have each to the other on the Globe. Hence he demonstrates that the Sum of all the Secants between any two Latitudes, divided by Radius, is the length of the Meridian Line contained between these two Latitudes, and made his Table of Meridional parts by the perpetual

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Addition of the Natural Secants of every whole minute, beginning with the Secant of one minute, and ending with the Secant of 89, 59, which being done, he finds fault with it, and saith it was something too great.

Then seeing Mr. *Wright* found fault with his own Table, that Learned Mathematician Mr. *William Oughtred* took it into Examination; and concluded that the true Sum of the Secants divided by Radius, is the length of the Meridian line, the finding of which he long endeavoured, but never attained it. Then he thought that if a minute were divided into Ten thousand equal parts, and a Table of Natural Secants made to all those minute parts, then those Secants being added together, their Sum (as he concluded) would be so near the true Sum of the infinite Secants, that no sensible errors could insue (but never considered, that though a Table of Natural Secants should be made to the least imaginable parts of a Circle, that the Sum of those Secants made by Addition, would exceed the true Area of the Secants space (which is the true Sum of those infinite Secants he sought for) by more than half the Sum of the first difference of all the Secants of those Minute Parts (which I shall demonstrate in this Paper) but seeing the making of such a Table of Natural Secants would be extreamly tedious, therefore he desisted with this Consideration, that the decrease would happen in the Decimal parts, remote from degrees.

Moreover, though Mr. *Oughtred* gave over thinking that the Area of the Secants space was past finding out, yet Mr. *John Collins* continued in the Inquiry, and sought the Solution of that Problem himself, and communicated it to all others, such as he thought were or would be capable of Solving of Problems of that Nature. In the Year 1666 Mr. *Collins* communicated it to me, and told me that if the true Sum of the Secants were found, we should have the true length of the Meridian line, on *Mercators Chart*.

Again in the Year 1668, the said Mr. *Collins* Communicated this Problem to *James Gregorie* (of *Aberdeen* in *Scotland*) who came and was Resident in *London* some months in his return from *Padoua*, where he writ the true Quadrature of the Circle and Hyperbola, his (*Geometriæ pars Universalis*, and his *Opticks*) who solved this Problem, and Printed it the same year in *London*, in his *Exercitationes Geometricæ*, in which after a long and intricate Demonstration, to prove that the Meridian line on the Planisphere, *est Scala Logarithmorum Excessuum, quibus secantes Latitudinum superant earundem tangentes posito radio loco unita*. He gives us to find the whole Sum of the Secants, this Rule, As the Radius is to $\frac{1}{4}$ of the Periphery of a Circle: So is the Square of Radius to the whole Sum of the Secants; in Fol. 19, and sendeth us to the Second Proposition of his Geometrical pars *Universalis*, for its Demonstration. Now he that works by this and finds the Sum of the Secants, and thence to find the length of the Meridian-

line,

line, divideth it by the Radius, shall find in the Quotient no more nor less than the $\frac{1}{4}$ of of the Periphery, which I Demonstrate thus: put the Radius $= a$ and $\frac{1}{4}$ of the Periphery $= b$, then it is, As $a . b :: aa . \frac{baa}{a} = ba$ and $\frac{ba}{a} = b = \frac{1}{4}$ the Periph. which was to be demonstrated.

To find the Sum of the Secants of any Arch less than $\frac{1}{4}$ of the Periph. his Rule is thus, As the Sine of the Arch is to the length of the Arch, so is the Rect-Angle made of the Sine of the Arch and the Radius, to the Sum of the Secants of that Arch; which divided by Radius, the Quotient will be = to the length of the given Arch, as in the last. Put the Right-sine of the Arch $= a$ the length of the Arch $= b$, and the Radius $= d$; now the Demonstration will be as followeth,

As $a . b :: \frac{adb}{a} = db$. and $\frac{db}{d} = b =$ the length of the Arch Q E.

Soon after Mr. *Gregories* Book was Published, Mr. *Collin's* desired Dr. *Barrow* to take the Problem of finding the true Sum of the Secants into his consideration, which he did, and verified what Mr. *Gregory* had published before false, with a clear *Mathematical Demonstration*, which is the Third Proposition of the *Appendix* to his 12th *Lecture fol.* 111, and 166th Figure.

The reason why both those able *Mathematicians* gave a false Solution of this Problem, was because neither of them did understand what kind of Figure that was whose Area they sought, which appears by their own Figures; from whence it is plain that both of them did force all those Secants that do pass from the Center through every of the least imaginable parts contained in a Quadrant, to stand perpendicular upon the length of the Radius, which is impossible: for how can the length 1571 stand at length upon the length of the Radius 1000 without folding or standing double in some place or other on the Line: This is both their Cases, for they both confine the Secants of the whole Quadrant to stand perpendicular upon the Radius, whereas they must stand perpendicular upon $\frac{1}{4}$ of the Periphery of the whole Circle, then the Curve Line prescribed by the extremity of the Secants, is equal to the Tangent of the Arch; and the extream Secant is the Secant of the Arch, the other side of the Figure is the Radius.

This I shall Demonstrate, and thence shew how the true Area of this Figure is found, which is the true Sum of the Secants.

Construction, First describe the Semi-circle A B E D, which we do suppose to roul as a Wheel, till that the Point E falls in F, B in C, A in G, and D in K, in which motion the Point E descibes the Curve E r F, B the Curve B C, and D the Curve D o K. Now because the Semi-circle

A B E D is supposed to roul upon the Line B F, therefore B F is equal to the length of the ¼ part of the Periphery of a whole Circle = B E, and the Curve Line described by the Points E, B, and D, are Segments of the Cycloid, by Dr. *Wallis* on the Cycloid; next continue the Line B F infinitely towards R, then is F R a Tangent Line; now from G the Center, draw the Secants G *a b*, G *d l* and G r *x*, and imagine the space G F R G, which contains all those Secants that can possibly be, or imagined possible to be drawn from the Center G to the Tangent Line F R, to be drawn; then draw the right Sines *a c*, *d e*, and r *z*, and imagine all the rest to be drawn; this done, let the Semi-circle G C F K be supposed to roul upon the Tangent-Line F R until the Point *a*, in the Arch do fall in *a* on the Tangent-Line, *d* in *d*, and r in r, and also the Point K in I; then is F I = in length to the Quadrant F K. Now because that by the 16th of the Third of *Euclid*, All Lines drawn from the Center of a Circle, and extended beyond the Circle, that Line cuts the Periphery of that Circle at Right-Angles; therefore the Secants G *a b*, G *d l*, and G r *x*, do cut the Periphery F K at Right-Angles in *a*, *d*, and r; whence it is plain that when the Semi-circle G C F K is roul'd so far on the Tangent-Line F R, that the Point *a* in the Arch cometh to fall on the Tangent Line in *a*, then will the Center G be the second G towards the left-hand, and the Secant of the Arch F *a*; that is, the Line G *a b* shall stand perpendicular upon the Line G G. Again, when the Semi-circle G C F K is roul'd so far on the Tangent-Line F R, that the Point *d* in the Arch falls on the Tangent-Line in *d*, then will the Center G be the third G, and the Secant G *d l* will stand perpendicular upon the Line G G. Thirdly, when the Semi-circle G C F K is roul'd so far on the Tangent-Line, that the Point r in the Arch falls on the Tangent-Line in r, then will the Center G be the fourth G, and the Secant G r *x* will stand perpendicular upon the Line G G. Fourthly and Lastly, when the Semi-circle G C F K is roul'd so far that the Point K falls in I, then will the Center G be last towards the left hand; now because K is the last Point of the Quadrant, C falls in H, and F in L, therefore the Secant of this Arch G I R is an infinite Line. Thus have I not only shewed how by the motion of the Semi-circle G C F K to set the whole Sum of the Secants which do naturally issue from G the Center of the Circle, and terminate at the Tangent-Line F R, perpendicular upon the Base G G, whose length is = F K the ¼ of the Periphery of the Circle, and made of a Triangular Figure of the Secants, *viz.* G F R, the four Lined Figure, *viz.* F G = Radius, G G = F K = the ¼ of the Periphery of the Circle. G R the extream Secant of the Quadrant F K, and the Curve Line F R = the Tangent Line F R; the Area of this Figure F G G R F is the true Sum of the infinite Secants required.

yt r

But

But in order to the finding of the true Sum of the Secants of any Arch, I shall first shew what kind of Figures the true Sums of the Right and Versed Sines do make, and also shew how to find the true Sum of the Right and Versed Sines, and also to find the true Sums of the Right and and Versed Sines of any Arch, and then proceed to the Operation.

First, When the Semi-circle GCFK is roul'd so far, that the Point *a* in the Arch do fall in the Point *a* in the Tangent-Line, F*R* then is *ac* the Right Sine of the Arch K*a* laid on the Secant G*ab* from *a* downwards to *c*.

Secondly, When the Semi-circle GCFK is roul'd so far that the Point *d* in the Arch doth fall in the Point *d* in the Tangent-Line *F R* then is *de* the Right Sine of the Arch K*d*, laid on the Secant *Gdl* from *d* to *e*.

Thirdly, when the Semi-circle GCFK is roul'd so far that the Point r in the Arch doth fall in r on the Tangent-Line F R, then is *z* r laid on the Secant G r *x* from r to *z*, and thus by this motion all the Sum of the Right Sines of the Quadrant FK are imagined to be set perpendicular downward on the Quadrant FK extended at length = FI, which terminates in the Curve Line *Gcez I*; therefore the Area of the Figure *FGcezIF* is equal to the three Lined Figure D*o*KED = the Square of Radius = the Sum of the Right Sine, by the first Corollary of the third Proposition of *Honorato Fabrio Opusculum Geometricum*. Now the Right Sine *ae* is the Sine of the Arch *Ka*; therefore *ac* is the Sine Complement of the Arch *Fa*, and the Right Sine *de* is the Sine of the Arch *Kd*; therefore *de* is the Sine Complement of the Arch *Fd*, and the Right Sine r*z* is the Sine of the Arch *K*r; therefore z r is the Sine Complement of the Arch *F*r, which is to be imagined of all the rest. Moreover the Versed Sine of an Arch is the difference between the Sine Complement of that Arch and the Radius. Therefore

Ga = *GF* = Radius — *ac* = *Gc* = Versed Sine of the Arch *Fa*
and *Gd* = *GF* = Radius — *de* = *Ge* = Versed Sine of the Arch *Fd*
and *G*r = *GF* = Radius — r*z* = *Gz* = Versed Sine of the Arch *F*r

Which is to be imagined of all the rest; therefore the Figure *GcezIGG* is equal to the whole Sum of the Versed Sines.

Now to find the whole Sum of the Versed Sines.

Put the Radius GF = 10000000, then is FI = 15707913,
Then GF = 10000000 × FI = 15707913 = GFIGG = 157079130000000.

But

But it is evident by Construction, that *Gcez IFG* the whole Sum of the Right Sines + *GcezIGG* the Sum of the

Versed Sine is = Rect-Angle GFIGG = 157079130000000
From which Substract G IFG = 100000000000000 the Sum

of the Right Sine, whose Differen. is GIGG = 057079130000000 the whole Sum of the Versed Sines Q E.

Now to find the Sum of the Right Sines of any Arch desired; *Honorato Fabrii* hath by the 5th Proposition of his *Opusculum Geometricum* Demonstrated the following Rule. As the Radius is to the Versed Sine of an Arch :: so is the whole Sum of the Right Sines, to the Sum of the Right Sine of the Arch required.

Example.

In the Parallellogram GFIGG let there be given the whole Sum of the Right Sine *GcezIFG* = 100000000000000, the Versed Sine of 60 d. *eG* = 05000000, and the Radius GF = 10000000, to find the Sum of the Sine contained in the Arch of 60 d. 00 m. *Iacezl*.

The Operation.

As the Radius ——————————— GF = 10000000
Is to the Versed Sine of 60 d. 00 m. ———— *lG* = 05000000
So is the whole Sum of the Right Sines *GcezIFG* = 100000000000000
To the Sum of the Right Sine of 60 d. 00 m. *Iacezl* = 050000000000000

Here we are to Note, that by Construction the Rect-Angle made by the Radius and the Length of an Arch, is ever equal to the Sum of the Sine Complement of an Arch + the Sum of the Versed Sine of that Arch; therefore to find the Sum of the Sine Complements of the Arch *Fa* = 30 d. 00 m. it is *GcezIFG* = 100000000000000 — *Iacezl* = 050000000000000 = *FacGF* = 050000000000000 the Sum of the Sine Complements of the Arch *Fa* = 30 d. 00 m. which was desired.

Hence to find the Sum of the Versed Sines of the Arch F*a* 30 d. 00 m. *viz.* the Area of the Figure G*c*G, I multiply the length of the Arch *Fa* = 5235988 by the Radius GF = 10000000, that Product is equal to the Parallelogram *FaGGF* = 52359880000000 from which

FacGF

$FacGF =$ 0500000000000000 the Sum of the Sine Complement of the Arch $Fa =$ 30 d. 00 m. being substracted, the Remainer will be GcG = 2359880000000 the Sum of the Versed Sine of the Arch Fa = 30 d. 00 m.

Corollary.

The Rect-Angle made by Radius, and the Right Sine of an Arch is equal to the Sum of the Sine Complement of that Arch.

The common Rule to find the Secant of an Arch by the Right Sines, is thus, As the Co-sine of an Arch, is to the Radius, so is the Radius to the Secant of that Arch: which stands by the Figure thus, As $ac : GF :: GF : Gb$. But better thus, put the Radius $GF = 1$, and the Versed Sine of an Arch $= a$, then the Sine Complement of an Arch will be $1 - a$; therefore it is as $1 - a : 1 :: 1 : \frac{1}{1-a}$ which being divided, the Quotient will be $1 + a + a^2 + a^3 + a^4 + a^5 + a^6 + a^7 + a^8 + a^9 + a^{10} + a^{11}$, &c. the Secant of the First Part upon the Arch.

Whence its manifest the Secant of an Arch is a Series of continual Proportionals, whose Termination is in the Curve Line $FblxR$; which by the Figure is as Gr Radius, is to $r3$ the Versed Sine of the Arch, so is $r3$ to 3.7 the fourth proportional: And as $r3 : 3.7 :: 3.7 : 7.8$. And again, as $3.7 : 7.8 :: 7.8 : 8.2$, &c. whose Termination is in x; the Sum of the Secants are found by adding of the first to the second, the second to the third, as followeth,

$$\begin{array}{llllllll} \text{The first} & = 1 & + a & + a^2 & + a^3 & + a^4 & + a^5 & \\ \text{The second} & = 1 & + 2a & + 4a^2 & + 8a^3 & + 16a^4 & + 32a^5 & \left.\right\} \text{\&c.} = \\ \text{The third} & = 1 & + 3a & + 9a^2 & + 27a^3 & + 81a^4 & + 243a^5 & \\ \text{\&c.} & & & & & & & \\ \hline & = 3 & + 6a & + 14a^2 & + 36a^3 & + 98a^4 & + 276a^5 & \end{array}$$

Moreover, though the Secants space is not the very same with the Hyperbolick space, yet it is of the same nature, as it appeareth by the dividing of an Unite, by an Unite less than the quantity assigned. Therefore the Secant space falleth under an infinite Series compounded of Ingredients of the same nature, and as proper to this Problem as that of the Hyperbola; for in the Hyperbola we have a Square made of the Difference between the Divisor and an Unite, whose half is an equal sided Plain Triangle or half Square. But in this Problem the Difference between the Sine Complement of an Arch, (the Divisor) and the Radius is equal to the Versed Sine of that Arch, whose length is ever less than the length of the Arch to which it belongeth, in the place of half the Square in the Hyperbola.

perbola. I have a Right Angled Plain Triangle, whose Base is the length of an Arch, and the Perpendicular the Versed Sine belonging to that Arch, and the Hypotenuse is a Curve-Line which falls within a streight Line, the Area of this mix'd Right-Angled Plain Triangle is the Sum of the Versed Sine of the Arch on which they stand; this Sum of the Versed Sines, I call the $\frac{1}{2}P^2$ that is half of the second Power; I put P to signifie Power, because the Base is ever longer than the Perpendicular. Now to proceed, I double the Sum of the Versed Sines, which being doubled maketh a plain Parallellogram, which being multiplied by the Versed Sine of the Arch, that Product is a long Cube or Parallellopipedon, which I divide by 3, and that Quotient I call one third of the third Power, which stands thus $\frac{1}{3}P^3$; then this Parallelopipedon being multiplied by the aforesaid Versed Sine, and that Product divided by 4 that Quotient shall be the $\frac{1}{4}$ of the Squared Square, or of the fourth Power, which stand thus $\frac{1}{4}P^4$, thus by the continual Multiplication of the several Products by the said Versed Sine; and then dividing each Product by the Index of the Power, that is the third Product by 3; the fourth Product by 4, the fifth by 5, *&c.* which I put thus $\frac{1}{3}P^3$, $\frac{1}{4}P^4$, $\frac{1}{5}P^5$, $\frac{1}{6}P^6$, $\frac{1}{7}P^7$, $\frac{1}{8}P^8$, *&c.*

Moreover, for your better Understanding of the Solution of this Problem according to Dr. *Wallis* on the Quaderature of the Hyperbola.

$$\text{We put}\begin{cases} 1 + 1 + 1 \;\&c. = \text{Radius} \times \text{length of the Arch} & = A \\ a + 2a + 3a \;\&c. = \text{Sum of the Versed Sines} & = \frac{1}{2}P^2 \\ a^2 + 4a^2 + 9a^2 \;\&c. = \frac{1}{3}\ \text{Parallellopipedon} & = \frac{1}{3}P^3 \\ a^3 + 8a^3 + 27a^3 \;\&c. = \frac{1}{4}\ \text{of the Squared Square} & = \frac{1}{4}P^4 \\ a^4 + 16a^4 + 81a^4 \;\&c. = \frac{1}{5}\ \text{of the Surd Solid} & = \frac{1}{5}P^5 \end{cases}$$

Hence it is plain that the whole Sum of the Secants $GRxlbFG$ is equal to the infinite Sum of $A + \frac{1}{2}P^2 + \frac{1}{3}P^3 + \frac{1}{4}P^4 + \frac{1}{5}P^5$, *&c.*

Then for *Example*, Let it be required to find the Sum of the Secants of the Arch $Fa = 30$ d. 00 m. which is the Area of the space $GbFG$.

First, In order to the Operation, I multiply the length of the Arch $Fa = 5235988$ by the Radius $GF = 10000000$, whose Product is the Area of the Parallellogram $GaFG = A = 52359880000000$, which is the first set down of the Synopsis. Secondly, Set down the Sum of the Versed Sine $aiF = GcG = \frac{1}{2}P^2 = 2359880000000$ Unity, under Unity, that is 8 under 8. Thirdly, The Sum of the Versed Sines being doubled, and that Sum being multiplied by the Versed Sine of the Arch Fa, which is $a1 = Gc = 1339746$, and that Product divided by its proper Index 3, the Quotient will be $\frac{1}{3}P^3 = 210775986032$, *&c.* which place as in the Synopsis, one place short towards the Left-hand, then Multiply the last Product by the aforesaid Versed Sine $a1 = 1339746$, and that Product being divided

divided by its proper Index 4, the Quotient will be $\frac{1}{4}P^4 = 211789713 12$, &c. which set one place short towards the left-hand, and then multiply the last Product by the aforesaid Versed Sine $a\,1 = Gc = 1339746$, and that Product being divided by its proper Index 5, the Quotient will be $\frac{1}{5}P^5 = 2699553162$, &c. which being placed as aforesaid, and continuing thus multiplying still the last Product by the same Versed Sine, and dividing every Product by its proper Index, ever placing that Quotient as aforesaid, until you arrive directly under the last place of Decimals assigned with one Figure, as in the *Synopsis*; this being done, I by Addition did find the true Area of the space $GbFG = 5495467.1242579$, which is the true Sum of the infinite Secants contained in the Arch $Fa = 30$ d. 00 m. which being divided by Radius, the Quotient is 5495467, and because that the Decimal Fraction 1242579 is much less than an Unite, therefore I neglect it as of no value, and I divide 5495467 by this improper Fraction $\frac{628211852}{360}$ the Quotient is $31\frac{10580687}{628318531}$, which being reduced into a Decimal Fraction, the true length of the Meridian-line from the Æquinoctial, to the parallel of 30 d. 00 m. is $= 31\frac{168}{1000}$ Æquinoctial Degrees, according to Mr. *Oughtred*.

The *Synopsis*.

The Right Angled Figure $GFaG$	$= A$	$= 5235988.0000000$
The Sum of the Versed Sines $a\,1F$	$= \frac{1}{2}P^2$	$= 0235988.0000000$
	$\frac{1}{3}P^3$	$= 0021077.5986032$
	$\frac{1}{4}P^4$	$= 0002117.8971312$
	$\frac{1}{5}P^5$	$= 0000269.9553162$
	$\frac{1}{6}P^6$	$= 0000025.2430297$
	$\frac{1}{7}P^7$	$= 0000002.9010277$
	$\frac{1}{8}P^8$	$= 0000000.3411651$
	$\frac{1}{9}P^9$	$= 0000000.0406291$
	$\frac{1}{10}P^{10}$	$= 0000000.0048979$
	$\frac{1}{11}P^{11}$	$= 0000000.0005963$
	$\frac{1}{12}P^{12}$	$= 0000000.0000733$
	$\frac{1}{13}P^{13}$	$= 0000000.0000091$
	$\frac{1}{14}P^{14}$	$= 0000000.0000001$

The true Area of the Figure $FbGG = 5495467.1242579$

Which reduced into degrees, makes 31 d. $\frac{168}{1000}$ And in this manner I found the true length of the Meridian-line from the Equator, to the Parallel of 50 d. 00 m. to be equal $10091347.0241352 = 57$ d. $\frac{812}{1000}$. And for 60 d. 00 m. to be equal $12919708.7579278 = 74\frac{022}{1000}$. And 70 d. 00 m. $= 87\frac{208}{1000}$. And for 80 d. 00 m. $= 104\frac{217}{1000}$. And for 89 d. 59 m. $= 125.464$, which is the true length of the Meridian-line, from the Æquinoctial to the

Parallel of 89 d. 59 m. which is not so much as ¼ of the length of the Meridian-line made by the *Addition* of the *Secants*, which difference ariseth hence, for the sum of the *Secants* made by the Addition of the *Secants* of any *Arch*, though a minute be divided into Ten thousand parts, shall exceed the true Area of the *Secants* space, or the true Sum of the *Infinite Secants* of any *Arch*, by more than half the Sum of the first differences of the *Secants* of every one of the *Secants* of those minute parts.

Let *B x* represent the length of an *Arch* extended, which let be divided into equal parts, *B C, C D, D E, E F, F G, &c.* each part containing one minute, or any minute part of the Circumference of a Circle, and upon this line, and from each point *B, C, D, E, F* and *G*, raise perpendicular-lines, as *B A* = Radius, *C c* = *Secant* of one part, *D f* = the *Secant* of two parts, *E* 1 = *Secant* of three parts, *F m* = *Secant* of four parts, *G p* = *Secant* of five parts, *&c.* Then through the Points *A, c, f,* 1, *m* and *p*, draw the Curve-line *A, c, f,* 1, *m* and *p*, and parallel to the line *B x*, draw the lines *A z, c b, c e, f d, f b, G i*, and compleat the Parallellograms *z b, e d, h g, &c.* Now I say that the right Angled Figure *A z c b A* is = difference between the Radius *A B* and the *Secant* of the first minute *C c*, and the right Angled Figure *c e f d* = difference between the *Secant* of the first minute *C c*, and the *Secant* of the second minute *D f*, &c.

Demonstration. Forasmuch as BC = CD = DE = EF = FG one part, therefore the Parallellogram BCZAB = Radius BA; and by the same reason the Parallellogram BCcbB is equal to the Secant of the first minute part Cc, therefore BCcbB — BCZAB = AzcbA the difference between BA the Radius, and Cc the Secant of the first minute part. Then Dd — Dc = Cb = the difference between Cc the Secant of the first minute part, and Df the Secant of the second minute part, then Eg — EF = Dd = hg the difference between Df and Ei, then Fk — Fi = Eg = lk, the difference between Ei and Fm: And then Gn — Gm = Fk = on, the difference between Fm and Gp, *&c.* Hence it is evident that the Sum of the Secant made by the aggregate of the Natural Secant, will be Cb + Dd + Eg + Fk + Gn, *&c.* whose Sum is the Figure BGpnmkigfdcbB, whereas the true Sum of these Secants is really the space BGpwmuitfrcsAB, Therefore the true Sum of the Secant of the Arch BG is equal to the Figure BGpnmkigfdcbB — space ABsc + cdrf + fgti + kum + mnwp; wherefore the Sum of the Secant made by the Addition of the Secants of any Arch, though a minute be divided into never so many parts, shall exceed the true Area of the Secant space, or the true Sum of the Infinite Secant of an Arch, by more than half the Sum of the first difference of the Secant of every one of the Secants of those minute parts, Q. E.

Thus

Thus have I found and shall demonstrate (else where) that the Sum of the Secants, however attained, hath no relation to the Meridian-line on the true Sea Chart.

In order to which, I shall resolve the following Problem, Admit a Ship to be at A under the Æquinoctial, and is bound to an Island B in the Latitude of 89 d. 59 m. whose Longitude to the Westward is CB=70d.00 m. the Question is, On what Rumb the Ship must Sail from A to B? First by the plain Sea Chart, secondly, by the length of the Meridian-line, which I have here found to be 125.464, thirdly, by the length of the Meridian line which Mr. *Wright* made by the Addition of the Secants of whole minutes, which is 539+. The Question truly Resolved, The true Rumb on which the Ship must Sail from A to B by the Plain Sea Chart is North 37d. 53 m. West; by the second she must Sail from A to B North 29 d. 09 m. Westerly; by the third, which according to Mr. *Wright* or *Mercator*, she must Sail from A to B North 07 d. 24 m. Westerly,

But that Ship which Sailes from *A* intending to arrive at *B*, by either of these three Rumbs, shall as certainly fail of their Desires, as he did who indeavoured to draw the proportion of *Hercules* Body by his foot-steps on the sand. For I say, That Ship which sails from A intending to find B, must sail directly North, until she is in the Latitude 89 d. 59 m. then shall B bear West from the Ship distant 1.22 mile, which may be seen that distance, if but four foot above the Water.

Demonstration. Seeing that the Parallel of 89 d. 59 m. wanteth but one mile of the Pole, and that mile being made the Radius of that Parallel, then shall the Circumference of that Circle be 6.28 miles: Then if 360 Degrees of Longitude be contained in 6.28 miles, in how many miles is 70 d. of Longitude contained, which by multiplying and dividing according to the Rule of Three, I find 1.22 answering to 70 d. of Longitude in the Æquinoctial Circle. Therefore that Ship which is to sail from *A* under the Æquinoctial to *B* in 89 d. 59 m. North Latitude, whose Longitude to the Westward is 70 d. 00 m. must sail North, until she is in the Latitude of 89d. 56m. then shall *B* bear West from her distant 1.22 mile, *Q. E.* The reason why I took this extream Problem, To demonstrate in part to the World, That Mr. *Wright*'s Projection is false, was, because that the truth of this Problem is evident meerly by inspection on the Globe: Thus much for this single sheet. But I have already written in order to Navigation fifty sheets of paper, to which I shall, God willing, add the manner of describing of a *Spherical Superficies* into a Concave Cylinder Geometrically of any given length greater than the Diameter of the *Sphere*; provided their Diameters be equal; this was Mr *Wright*'s Case.

More

Moreover, I shall Geometrically shew the Description of *Spherical Superficies* into a Concave Cylinder, their Diameters agreeing, in such manner that in all its parts it shall Mathematically agree with the Globe, and that the Meridian-line hath no relation to the *Sum* of the *Secant* divided by Radius, as was asserted by Mr. *Wright* and others.

And also I shall shew the making and use of the true *Sea Chart*, and shall Problematically demonstrate the plain *Sea Chart* to be false.

And shall shew the difference between the distance between two places in the Arch of a great Circle, and the distance in the Rumb-line, on which a *Ship* must sail from one place to another.

And shall shew the difference between the Angles of Position and the Rumb-line on the Globe.

And shall shew how to keep a perfect Journal in such manner, as that any man which is capable of taking Charge of a *Ship* at *Sea*, may judge whether the Reckoning were well kept or not.

I shall also annex an Examination, which all such Navigators ought to pass, that shall be judged capable of taking Charge as Master of any of his Majesties or Merchants Ships, *&c.*

Query. Whether or not, it be possible Mathematically to demonstrate, how deep a *Ship* must go in the water, and what difference there must be between her Draught of Water at her Head and Stern to sail best.

F I N I S.

London; Printed by *Tho. James* for the Author 1685.

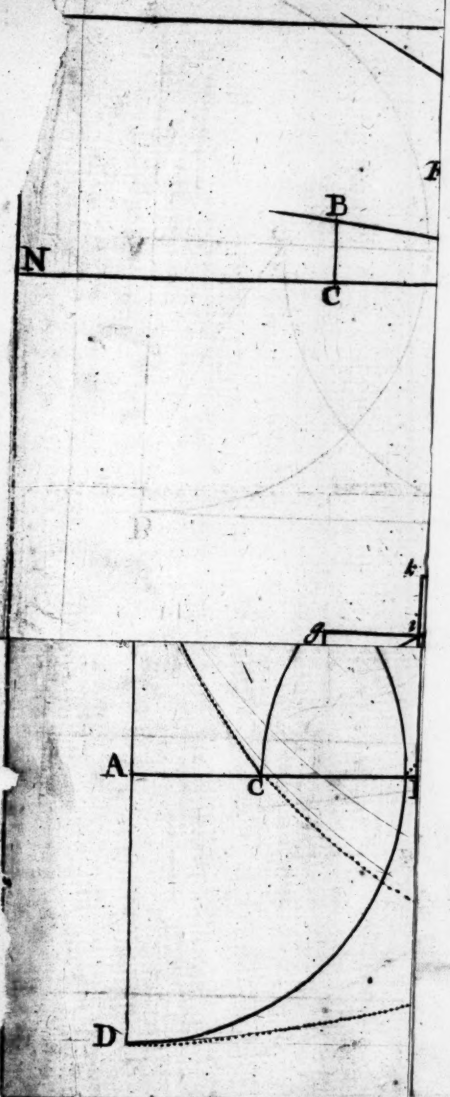
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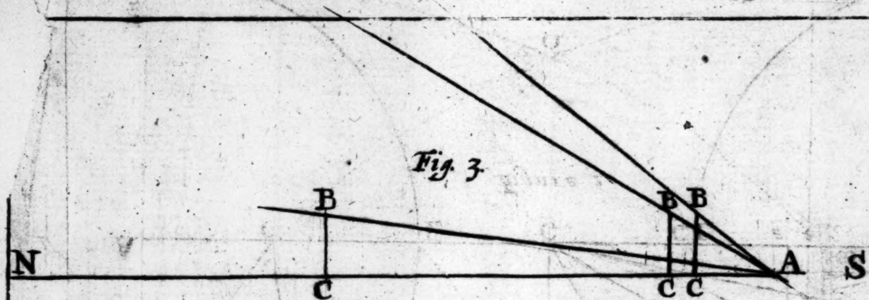
Fig. 3.



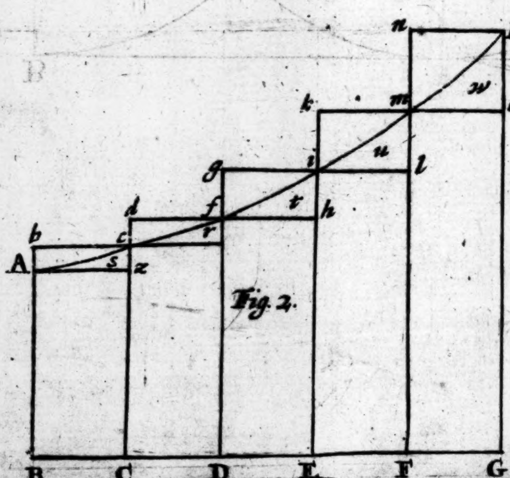
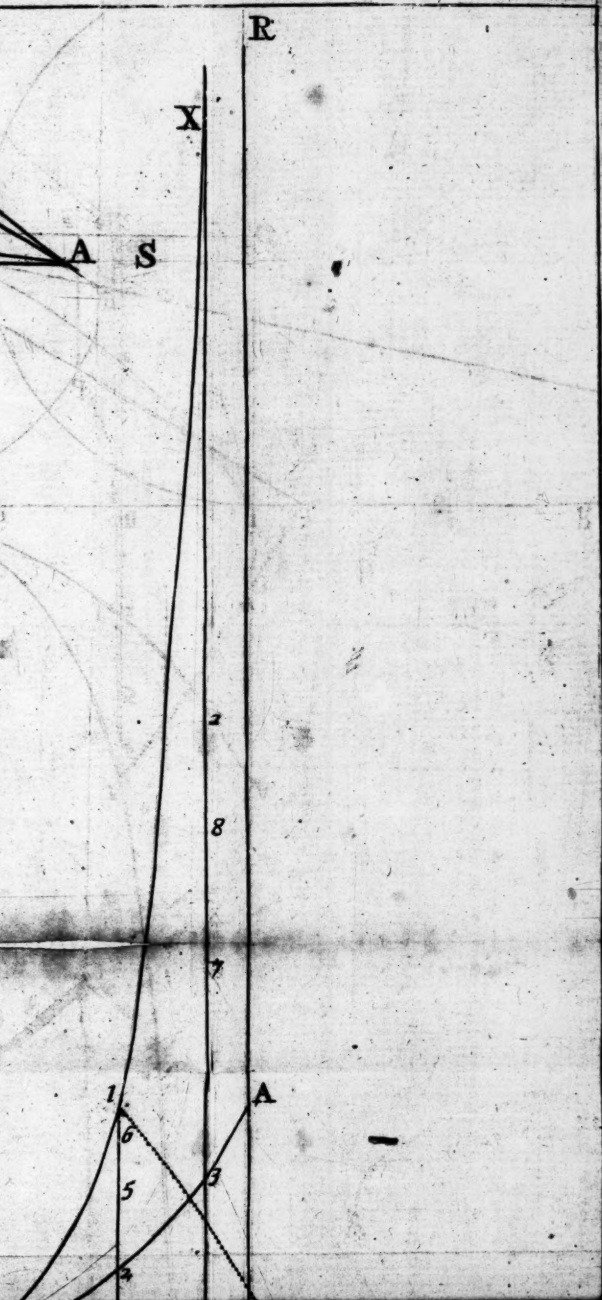


Fig. 2.



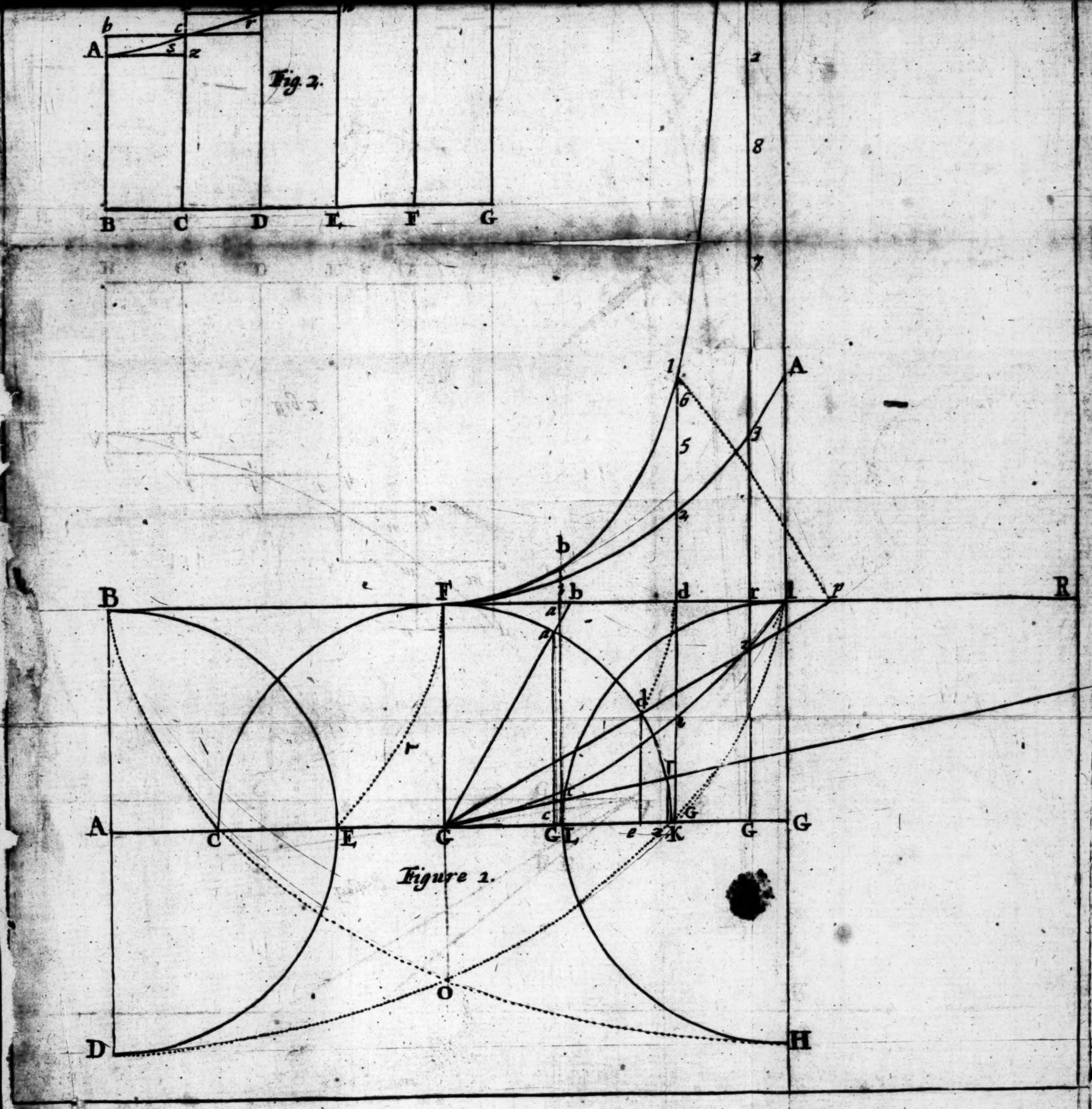


Fig. 2.

Figure 1.